$\Delta G(x)$
The first Milestone of RHIC Spin Program

Jianwei Qiu
Brookhaven National Laboratory

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Outline

- Compelling questions in spin physics?
- Existing Measurements and the QCD global analysis
- Is the ΔG or $\Delta G(x)$ really small?
- What is the consequence if $\Delta G \sim 0$?
- What could be done to better determine ΔG?

  – experimentally vs theoretically
- Summary and outlook

Compelling questions in spin physics

- ❑ **Research plan for spin physics at RHIC – 2005:**
  - ✧ How do gluons contribute to the proton spin?
  - ✧ What are the patterns of up, down, and strange quark and antiquark polarizations?
  - ✧ What orbital angular momenta do partons carry?
  - ✧ What is the role of transverse spin in QCD?
- ❑ **Status and Prospects – 2007, Plans – 2008:**
  - ✧ How do gluons contribute to the proton spin?
  - ✧ What is the flavor structure of the polarized sea in the nucleon?
  - ✧ What are the origins of transverse-spin phenomena in QCD?
- ❑ **The first milestone of RHIC spin program:**

  Determine the value of $\Delta G(x)$ or ΔG!

Why ΔG is so special?

- **So-called proton "Spin crisis":**

  In late 1980, EMC extended spin structure function $g_1(x)$ measurement to low x, and found that the net quark helicity contribution to proton's spin is only 10-20 %

  A contradiction to the Quark Model prediction – "spin crisis"

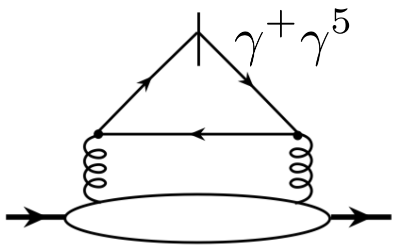
- **One possible solution to "Spin crisis":**

  Quark helicity: $\Delta q \propto \langle p,s|\overline{\psi}(0)\gamma^+\gamma^5\psi(0)|p,s\rangle$

  Total quark helicity: $\Sigma(Q^2) = \sum_q \left[\Delta q(Q^2) + \Delta\bar{q}(Q^2)\right]$

  Measured quark helicity: $\Sigma(Q^2) = \Sigma(Q^2)_{\rm true} - N_f \dfrac{\alpha_s(Q^2)}{2\pi}\Delta G(Q)$

  Anomaly contribution with a large ΔG cancels the true quark helicity



Proton spin in QCD

- **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk}\int d^3x\ M^{0jk}_{\rm QCD} \quad \longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\,x^\mu - T^{\alpha\mu}_{\rm QCD}\,x^\nu$$

Angular momentum density

- **Quark angular momentum operator:**

$$\vec{J}_q = \int d^3x\left[\psi_q^\dagger\vec{\gamma}\gamma_5\psi_q + \psi_q^\dagger(\vec{x}\times(-i\vec{D}))\,\psi_q\right]$$

- **Gluon angular momentum operator:**

$$\vec{J}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- **Proton spin and Ji's sum rule:**

$$S = \sum_f \langle P, S_z = 1/2|\,\hat{J}^z_f\,|P, S_z = 1/2\rangle = \frac{1}{2}$$

Gluon angular momentum

$$\frac{1}{2} = J_q(\mu^2) + J_g(\mu^2) = \left[\frac{1}{2}\Sigma(\mu^2) + L_q(\mu^2)\right] + J_g(\mu^2)$$

Gluon contribution to proton's spin

- **Gluon angular momentum:**

$$J_g(\mu^2) = \int d^3x \langle P,s|\vec{x}\times(\vec{E}\times\vec{B})|P,s\rangle \equiv \underbrace{\Delta G(\mu^2)}_{\text{Helicity}} + \underbrace{\left(J_g(\mu^2) - \Delta G(\mu^2)\right)}_{\text{Transverse motion}}$$

- **Asymptotic limit (Ji):**

$$J_q(\mu^2\to\infty) \to \frac{1}{2}\,\frac{3N_f}{16+3N_f} \qquad J_g(\mu^2\to\infty) \to \frac{1}{2}\,\frac{16}{16+3N_f}$$

If ΔG is very small, total gluon angular momentum comes from its transverse motion!

- **Gluon TMD parton distribution:**

$$f_{g/h\uparrow}(x,\mathbf{k}_\perp,\vec{s}) \equiv f_{g/h}(x,k_\perp) + f^{\text{Sivers}}_{g/h\uparrow}(x,k_\perp)\,\vec{s}\cdot(\hat{p}\times\hat{\mathbf{k}}_\perp)$$

$$f^{\text{Sivers}}_{g/h\uparrow}(x,k_\perp)^{\text{SIDIS}} = -f^{\text{Sivers}}_{g/h\uparrow}(x,k_\perp)^{\text{DY}}$$

Moment of Sivers function ⟹ three-gluon correlation functions

QCD global analysis - I

❑ **DSSV – 2008:** $\Delta f_j^{1,[x_{\min}-1]}$ at $Q^2 = 10\,\mathrm{GeV}^2$

	$x_{\min}=0$	$x_{\min}=0.001$	
	best fit	$\Delta\chi^2=1$	$\Delta\chi^2/\chi^2=2\%$
$\Delta u+\Delta\bar{u}$	0.813	$0.793\ ^{+0.011}_{-0.012}$	$0.793\ ^{+0.028}_{-0.034}$
$\Delta d+\Delta\bar{d}$	-0.458	$-0.416\ ^{+0.011}_{-0.009}$	$-0.416\ ^{+0.035}_{-0.025}$
$\Delta\bar{u}$	0.036	$0.028\ ^{+0.021}_{-0.020}$	$0.028\ ^{+0.059}_{-0.059}$
$\Delta\bar{d}$	-0.115	$-0.089\ ^{+0.029}_{-0.029}$	$-0.089\ ^{+0.090}_{-0.080}$
$\Delta\bar{s}$	-0.057	$-0.006\ ^{+0.010}_{-0.012}$	$-0.006\ ^{+0.028}_{-0.031}$
Δg	-0.084	$0.013\ ^{+0.106}_{-0.120}$	$0.013\ ^{+0.702}_{-0.314}$
$\Delta\Sigma$	0.242	$0.366\ ^{+0.015}_{-0.018}$	$0.366\ ^{+0.042}_{-0.062}$

❑ **Hirai-Kumano – 2008:**

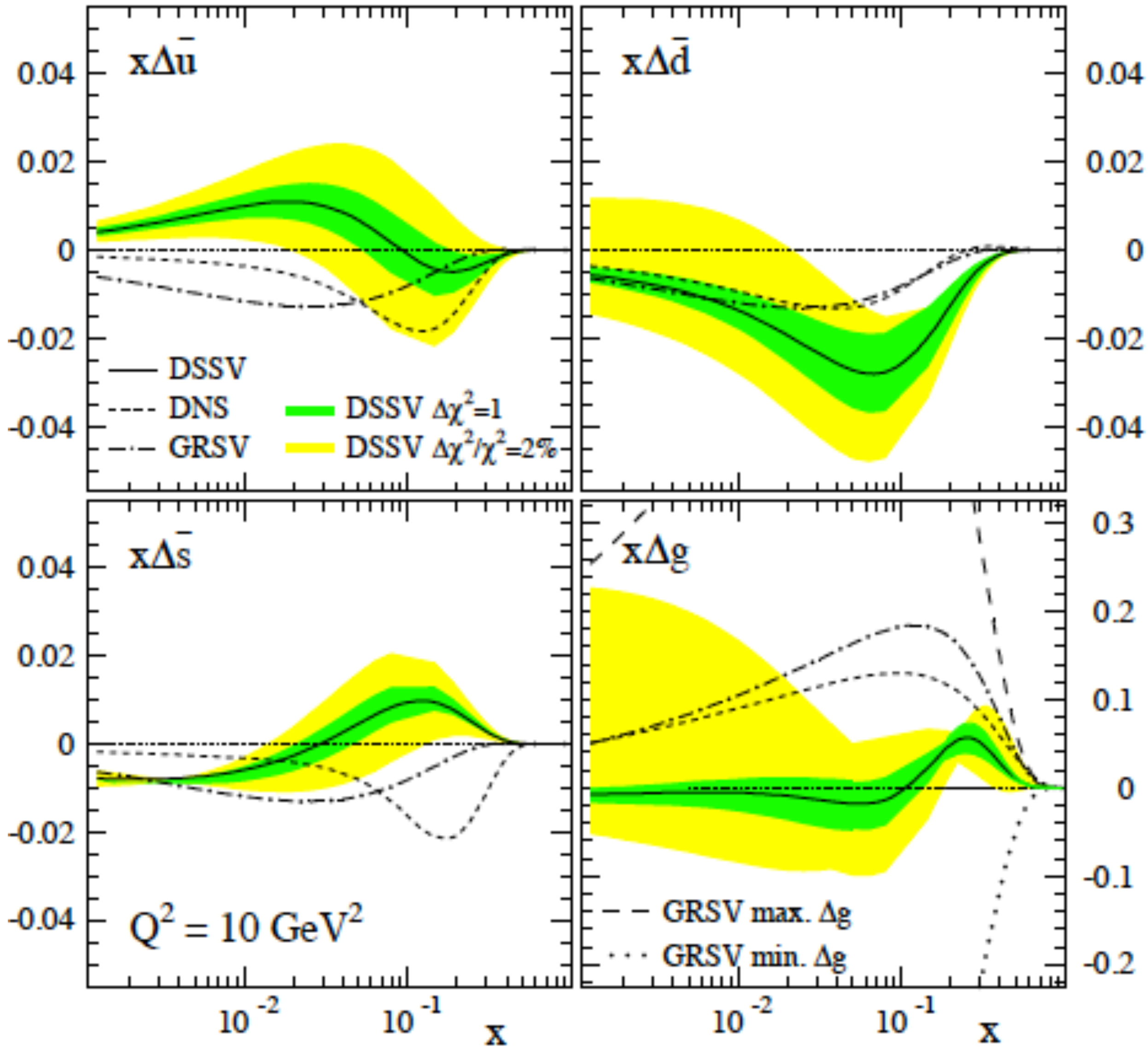
at $Q^2 = 1\ \mathrm{GeV}^2$

Analysis set	DIS	RHIC π^0	E07-011
A	◯	–	–
B	◯	◯	–
C	◯	–	◯

	Set A		Set B		Set C	
	Positive	Node	Positive	Node	Positive	Node
$\Delta\Sigma$	0.24 ± 0.07	0.22 ± 0.08	0.26 ± 0.06	0.25 ± 0.07	0.24 ± 0.05	0.22 ± 0.05
ΔG	0.63 ± 0.81	0.94 ± 1.66	0.40 ± 0.28	-0.12 ± 1.78	0.63 ± 0.45	0.94 ± 1.09

QCD global analysis - II

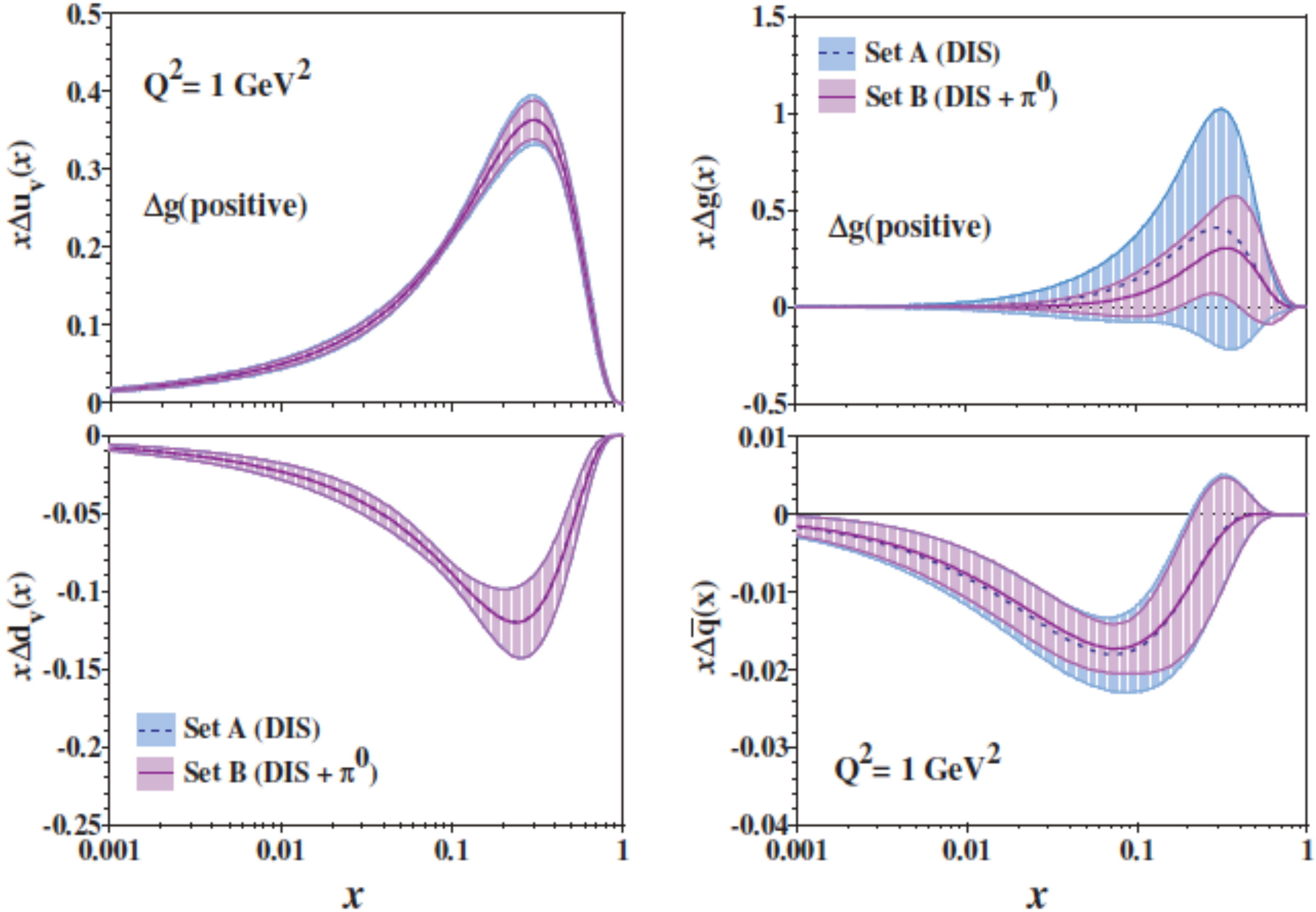
- DSSV:



Has a node

QCD global analysis - III

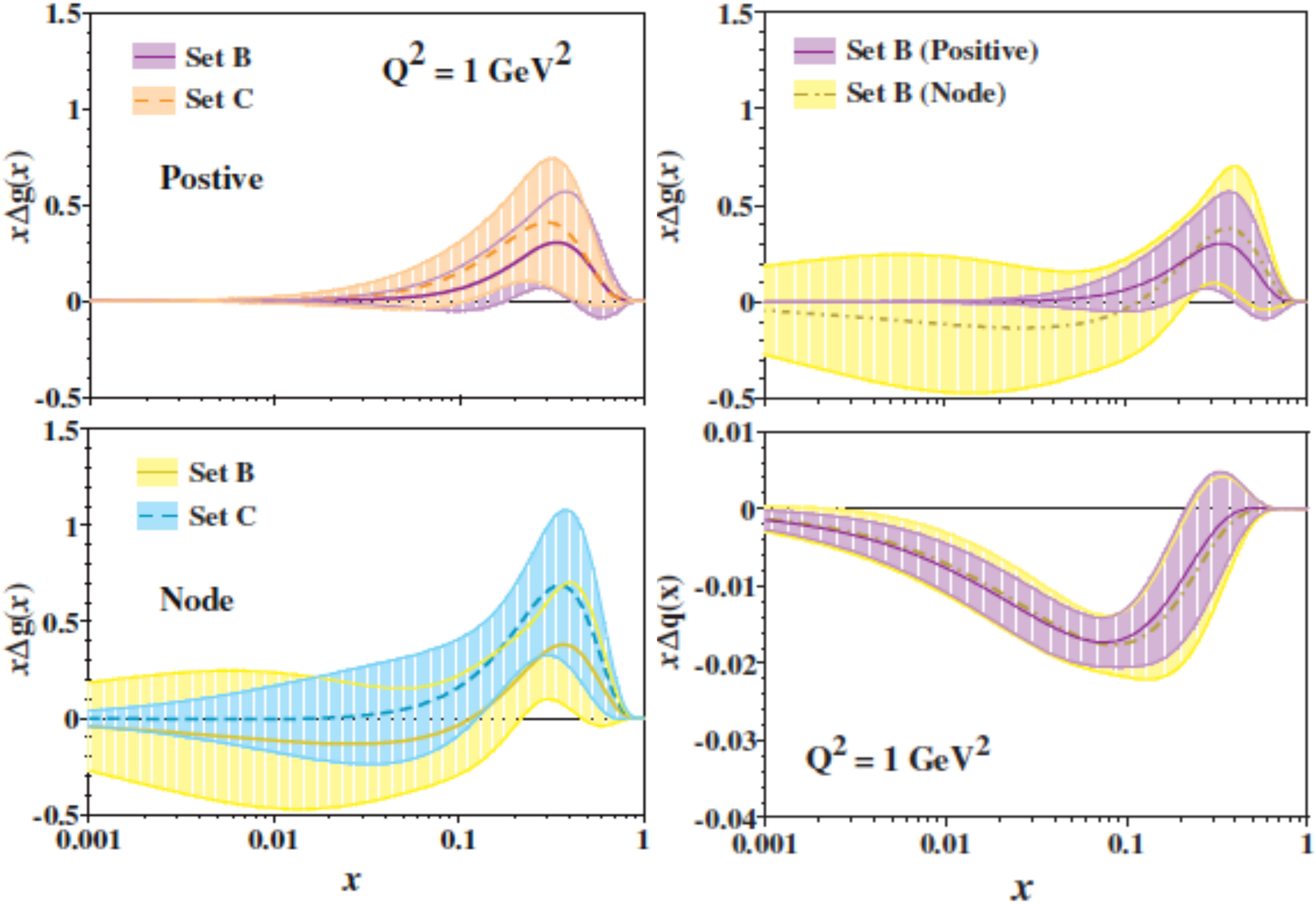
❑ **HK:**



Without a node!

QCD global analysis - III

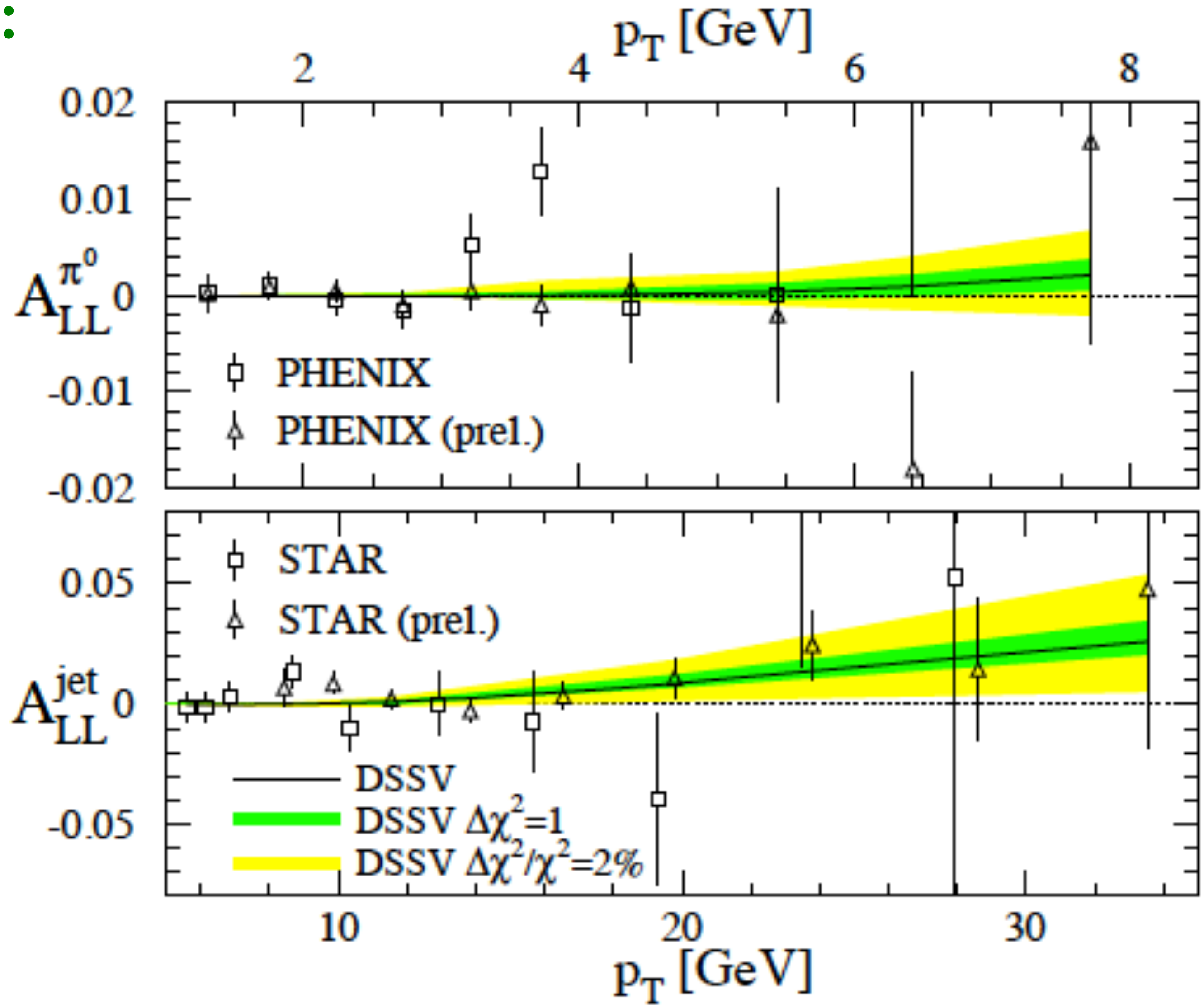
❑ **HK:**



With current data accuracy, large bias on the functional form

Comparison with RHIC data

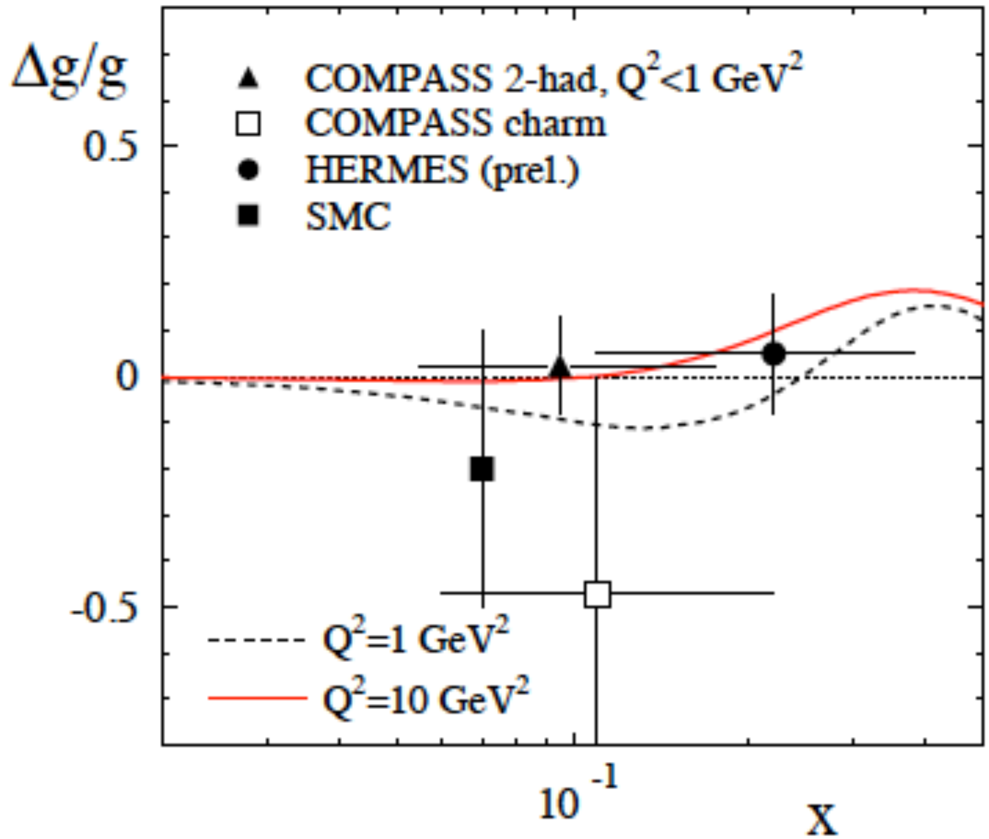
- ❑ **DSSV:**



Comparison with DIS data

- **DSSV:**

Data was not in the analysis



- **Observables:**

$$\ell p \to h + X \qquad \ell p \to h^+ + h^- + X$$

Current “conclusion(s)” on ΔG

- Anomalous gluonic contribution:

$$\Sigma(Q^2) = \Sigma(Q^2)_{\rm true} - N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q)$$

Need ΔG ~ 1.5 – 2, which is unlikely

- Gluonic contribution to proton spin:

$$J_g(\mu^2 \to \infty) \to \frac{1}{2} \frac{16}{16 + 3N_f} \sim \frac{1}{4}$$

“without gluon orbital”

Might be ruled out by DSSV based on RHIC data: $\Delta g = -0.084$

But, not by Hirai and Kumano yet:

	Set A		Set B		Set C	
	Positive	Node	Positive	Node	Positive	Node
$\Delta\Sigma$	0.24 ± 0.07	0.22 ± 0.08	0.26 ± 0.06	0.25 ± 0.07	0.24 ± 0.05	0.22 ± 0.05
ΔG	0.63 ± 0.81	0.94 ± 1.66	0.40 ± 0.28	-0.12 ± 1.78	0.63 ± 0.45	0.94 ± 1.09

Comments and questions

❑ Comments:

- ✧ Experiments measure hadronic or leptonic cross sections,
  NOT quark or gluon distribution functions or helicity distributions

- ✧ Size of collinear quark/gluon distributions or helicity distributions
  is "SCHEME" sensitive – artifact of collinear factorization
  – pQCD calculation is still consistent if F_2 has no gluon part

❑ Questions:

- ✧ Is getting a number for ΔG the goal of our ΔG program?

  we may never get the number to our satisfaction since the
  formula that we use to extract it does not valid for low and high x

- ✧ What about $\Delta G(x)$?

  It provides much richer information on QCD dynamics than ΔG
  It is the G(x), NOT the gluon momentum fraction G, got us excited

What the RHIC data try to tell us?

❑ The fact:

A_{LL} for inclusive jet or pion production at high p_T at RHIC is small

❑ Implication:

$\Delta G(x)$ is small in the x-range sensitive to the kinematics of data if one applies the leading twist pQCD factorization formula

❑ Cautions:

What data really says: the difference of two cross sections with spin flipped is much smaller than the cross section itself

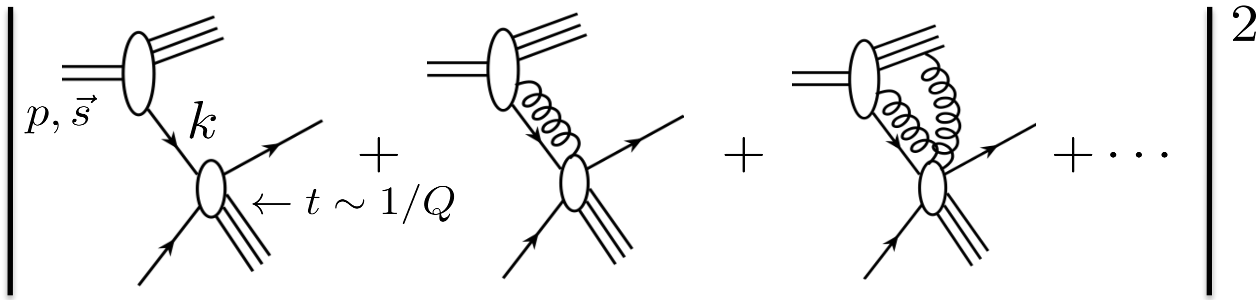
pQCD expression for the hadronic cross section is much more than the beautiful leading power factorized formula

$$d\sigma_{AB\to HX} = \sum_{abc} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to c} \otimes \phi_{c\to H} + \ldots$$

Hadronic cross sections in QCD

❑ Theorists' view of cross section:

$\sigma(Q,\vec{s}) \propto$



Any number of partons could participate in the collision

❑ Large momentum transfer simplifies the picture:

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

Single hard scale ➡ Leading power ➡ Collinear factorization

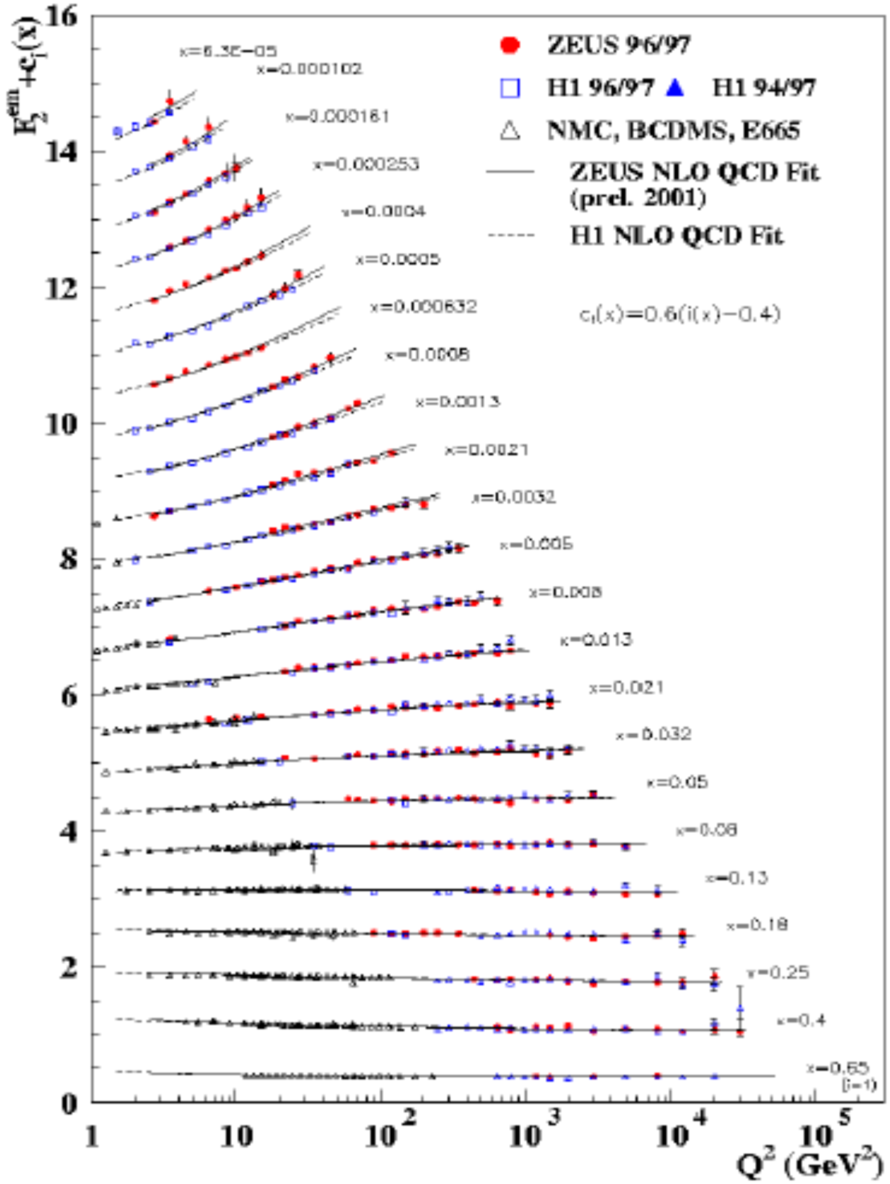
$$\sigma^{(2)}_{AB}(Q,\vec{s}) = \hat{\sigma}_{ab}(x,x',Q) \otimes f_{a/A}(x,Q,\vec{s}) \otimes \left[f_{b/B}(x',Q) \otimes \cdots\right]$$

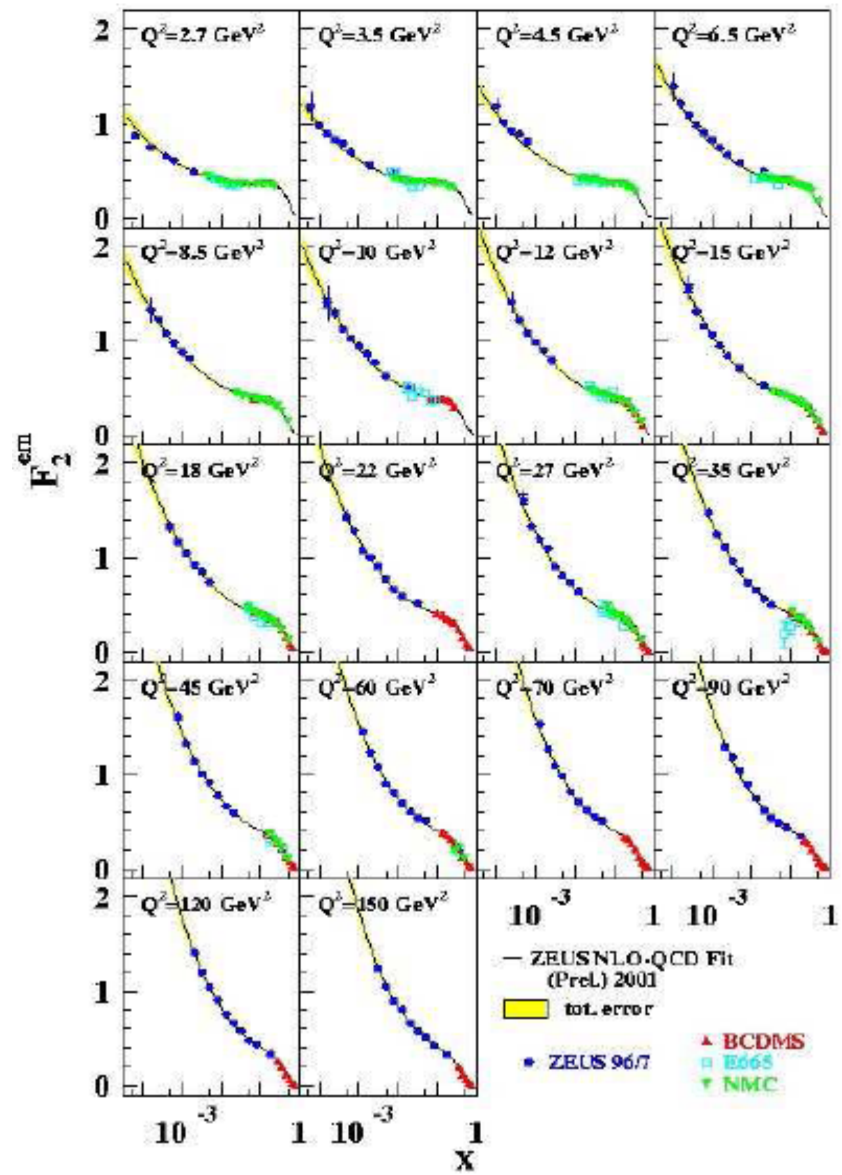
❑ Predictive power:

Short-distance dynamics, PDFs, and FFs

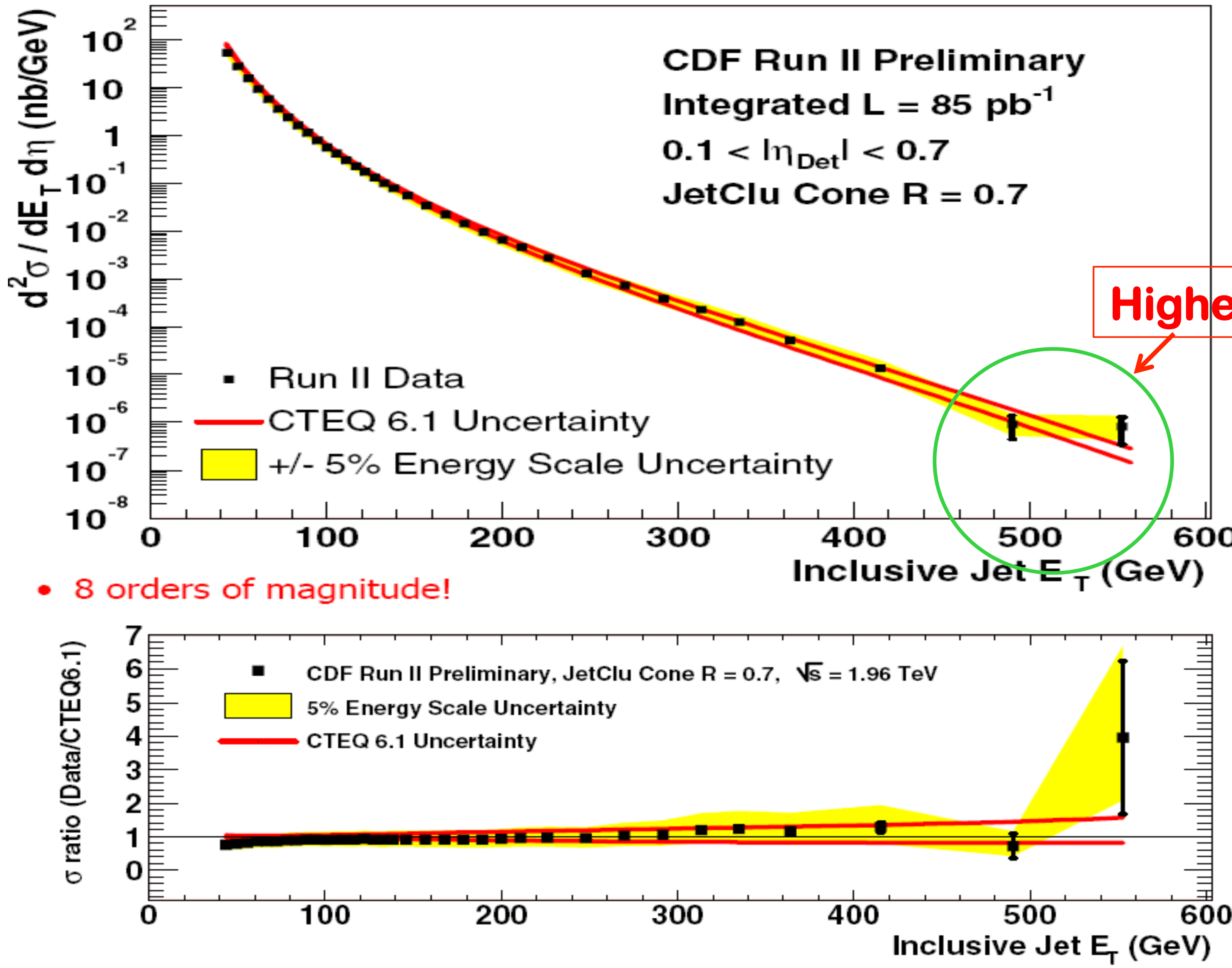
It worked beautifully – great success of QCD!

Leading power QCD vs DIS data





Leading power QCD vs hadronic jet data



Challenges

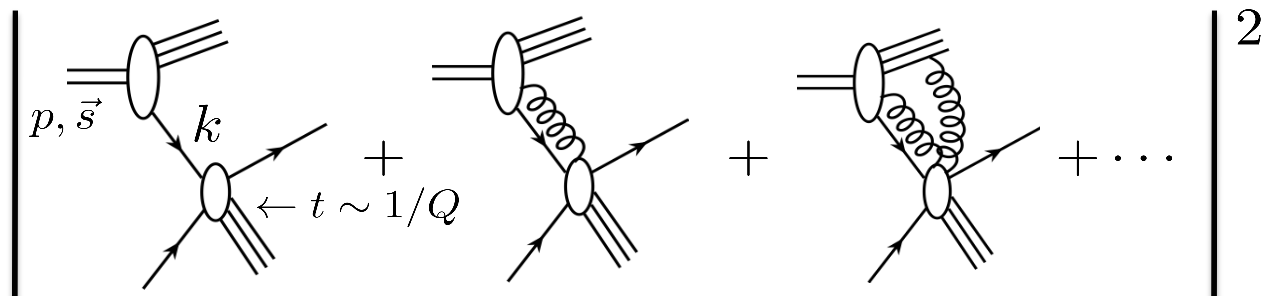
❑ Success of leading power (twist) QCD:

It is great for establishing QCD as the theory for strong interaction

It is also boring as far as nonperturbative QCD physics is concerned

❑ Moving beyond the local density?

$\sigma(Q, \vec{s}) \propto$



$$\sigma_{AB}(Q, \vec{s}) \approx \sigma_{AB}^{(2)}(Q, \vec{s}) + \frac{Q_s}{Q}\sigma_{AB}^{(3)}(Q, \vec{s}) + \frac{Q_s^2}{Q^2}\sigma_{AB}^{(4)}(Q, \vec{s}) + \cdots$$

Too large to compete?

✧ **Difference of cross sections - A_N: the leading power cancels**

✧ **Low-x physics: "long-range" coherence – every term is important**

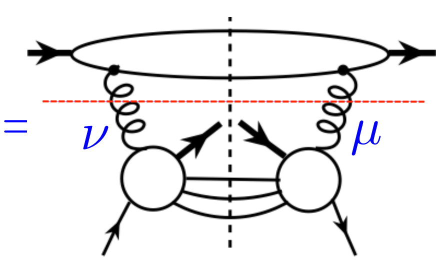
Another caution

Success of leading power calculation for a cross section does NOT guarantee a success for calculating the difference of two cross sections

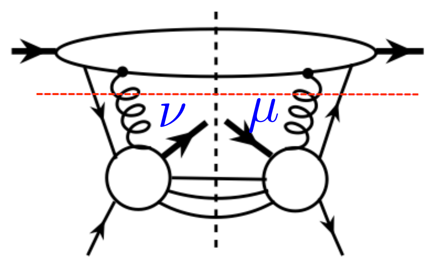
$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

When the leading power contribution is almost canceled, the subleading power contribution can no longer be neglected!

$d\Delta\sigma(p, s_{\|}) =$



$i\epsilon_{\mu\nu}$ $+$



$i\epsilon_{\mu\nu}\gamma^{\alpha}$

$d_{\mu\nu}\gamma^{\alpha}\gamma_5$

$+ \ldots$

Percent!

Could be of few % of cross section
But, not sure about the sign?
This is process dependent!

Possibilities

- Small A_{LL} of inclusive jet/pion production:

  Could indicates that $\Delta G(x)$ is small for the x-range

  How to confirm that?

$$A_{LL}|_{\text{jet/pion}} \propto \Delta G(x) \otimes \Delta G(x')$$

  Measure A_{LL} of observables having different dependence on $\Delta G(x)$

  Size of subleading power contribution is process dependent

- Possibilities at RHIC (of course at EIC as well):
  - ✧ Inclusive high p_T direct photon (or low mass Drell-Yan)
  - ✧ Photon – jet correlation
  - ✧ Charm to cover different x-range
  - ✧ J/ψ to have very different power corrections
  - ✧ ...

Opportunities

❑ Direct photon:

It is dominated by $q + g \rightarrow \gamma + q$ Compton subprocess, if $\Delta G(x)$ is sufficiently large

$$A_{LL} \propto g_1^p(x) \otimes \Delta G(x') + x \leftrightarrow x' + \ldots$$

We know something about this one

❑ J/ψ (My talk at RBRC workshop on CGC):

The production seems to be dominated by the heavy quark pair with axial vector charge for spin averaged production

Can this be held for the longitudinally polarized cross section or asymmetries?

Work assignments for the BNL Spin Summer Program!

Summary and outlook

- QCD global analysis of existing data indicates that ΔG is unlikely to be larger than 1.5 - 2

- However, existing data are not sufficient to say

  $\Delta G \ll 1/4$ The asymptotic value for gluon angular momentum

- Knowledge of ΔG(x) is clearly much more interesting and important than one number ΔG!

  It was the G(x), not momentum fraction G, that got people excited!

- RHIC Spin Program provides unique opportunities to explore QCD dynamics in a regime where we have not been able to do before, or near future

Thank you!

Backup slices